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Rolling down the throat in NS5-brane background: the case of electrified D-brane

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Abstract: We study rolling radion dynamics of electrified D-brane in NS5-brane background, both in effective field theory and in full open string theory. We construct exact boundary states and, from them, extract conserved Noether currents. We argue that T-duality and Lorentz boost offer an intuitive approach. In the limit of large number of NS5-branes, both boundary wave functions and conserved currents are sharply peaked and agree with those deduced from the effective field theory. As the number of NS5-branes is reduced, width around the peak becomes wider by string corrections. We also study radiative decay process. By applying Lorentz covariance, we show how the decay of electrified D-brane is related to that of bare D-brane. We compute spectral moments of final state energy and winding quantum number. Using Lorentz covariance argument, we explain in elementary way why winding quantum number should be included and derive rules how to do so. We conclude that Kutasov's "geometric realization" between radion rolling dynamics and tachyon rolling dynamics holds universally, both for bare and electrified D-branes.

Keywords: D-branes, Conformal Field Models in String Theory, Tachyon Condensation.

Contents

1. Introduction

An outstanding open problem in string theory is to formulate a framework for addressing string dynamics in time-dependent and cosmological backgrounds. By establishing such a framework, one hopes to understand better not only string theory itself but also conceptual issues in quantum gravity and quantum cosmology. Prompted by ground-breaking work of Sen [1], there has been considerable progress in understanding time-dependent phenomena in the open string theory: decay of unstable D-brane or brane-antibrane pair is describable by rolling and condensation of open string tachyon. Sen further argued [2] that, as the tachyon rolls down to potential minimum, an unstable D-brane or a brane-antibrane pair decays to a system consisting of pressureless dust called "tachyon matter" yet devoid of any obvious open string excitation.

Formation of non-threshold bound-state among branes is another situation that time-dependent string dynamics is involved. A well-known case is formation of a supersymmetric bound-state (p, q) string [3, 4] when a probe BPS D-string of charge $(p, 0)$ falls into a target BPS F-string of charge $(0, q)$ [5, 6]. Another case is formation of a supersymmetric bound-state of D-brane and NS5-brane [7] when a probe BPS D-brane falls into a target BPS NS5-brane [8]. The supergravity background of NS5-brane [9, 10] becomes in the near horizon limit an exactly solvable conformal field theory (CFT) [11, 12, 13], known as the "throat geometry". In both cases, radion rolling dynamics is describable in terms of the worldvolume theory in the background of target F-string or NS5-brane. Interestingly, in the worldvolume description, radion rolling dynamics of BPS D-branes resembles tachyon rolling dynamics of unstable D-branes or brane-antibrane pair. In particular, for appropriate background and regime of target NS5-branes, the works [8, 14] noted that the radion effective action takes exactly the same functional form as the tachyon effective action for unstable D-branes and, from this observation, proposed to view the radion rolling dynamics as a sort of "geometric realization" of tachyon rolling dynamics for unstable D-branes.

Is such "geometric realization" an artifact of low-energy effective approach or does it extend to full-fledged string theoretic analysis? This question was addressed in [15]. There, by utilizing the fact that the "throat geometry" is an exactly solvable CFT, boundary state of the rolling radion was constructed as a counterpart of boundary state for the rolling tachyon [16]. From them, positive conclusion was drawn that the "geometric realization" extends to the full string theory beyond the supergravity limit. See also [17].[1] More specifically, boundary state of the infalling D-brane was constructed by relating the CFT of the "throat geometry" [11, 12, 13] to $\mathcal{N}=2$ Liouville theory[2] and then by performing a suitable Wick rotation to so-called noncompact class-2 brane in the $\mathcal{N}=2$ Liouville theory [15]. The rolling radion boundary state constructed in [15] facilitated to study the dynamics exactly and to compare it with the dynamics of rolling tachyon for unstable D-brane. In particular, the investigation of decay rate to closed strings indicated that both rolling radion and rolling tachyon share exactly the same properties. Such universality is newly emergent feature that may lead to breakthroughs for better understanding of time-dependent phenomena in open string theory.

The purposes of this work are mainly twofold. First, this work aims at investigating aspects of the rolling radion boundary state constructed in [15] and to put it into a further consistency check. We shall do so by turning on homogeneous electric field on the worldvolume of the rolling Dp-brane in the direction parallel to the background NS5-brane worldvolume. This is the rolling radion counterpart to the rolling tachyon of unstable D-brane with worldvolume electric field [28, 29]. As emphasized in [29], electrifying a Dp-brane is equivalent in an appropriate limit to T-dual of boosting the D$(p-1)$-brane along the electric field direction. It is then intuitively clear that physical observables ought to transform covariantly under such Lorentz boost and T-duality map [30]. Adopting the same strategy as [29], we shall construct the boundary state of rolling radion with electric field, study decay rate to closed strings, and compare the result with that expected from intuitive approach based on Lorentz boost and T-duality map. Agreement between the two methods would serve as a consistency check that the boundary state constructed in [15] is indeed a correct one. Second, this work aims at studying whether and, if so, how F-strings of macroscopic size are formed as probe Dp-brane falls into the target NS5-brane. In an appropriate limit, the aforementioned Lorentz invariance argument asserts that such

[1] Related works can be found in [18]–[26].

[2] See [27] for review on the relation.

F-strings are certainly formed. An interesting question would then be details of spectral distribution of the formed F-strings. In fact, this issue is precisely the counterpart of F-string formation during tachyon rolling of unstable D-brane [29, 31, 32], an issue which was revisited in [33] in a different context.

Investigation of tachyon rolling for electrified D-brane was made precise in [28, 29] by constructing relevant boundary state and then comparing the dynamics against that deduced from effective field theory approach. As emphasized in [29], final decay product consists of fundamental strings in addition to "tachyon matter", a fact which again can be readily understood from the Lorentz invariance. We shall follow the same strategy and construct the boundary state of rolling radion for an electrified D-brane. We first study the Born-Infeld effective action of a rolling D-brane in the presence of electric field and obtain D-brane trajectory and energy-momentum tensor. Then we extend the analysis of [15] and construct the boundary states for the rolling D-brane in the presence of the constant electric field. Our construction of boundary states is based on the prescription proposed in [29] in the context of boundary states of the rolling tachyon in the presence of constant electric field. We will see that this prescription reproduces correct energy-momentum tensor and D-brane trajectory in the supergravity limit, $N \to \infty$. We also discuss quantum aspects by studying the closed string emission rate from the rolling D-brane. By applying the Lorentz boost and using Lorentz covariance of spectral observables, we prove in elementary way that winding quantum numbers are to be included. We find that universal feature of the decay process is recovered once we include winding modes.

This paper is organized as follows. In section 2, as a prelude, we study the effective field theory approach to radion rolling dynamics of electrified D-brane in the NS5-brane background. We obtain classical trajectory and conserved current tensors of the electrified D-brane. In section 3, we propose the radion rolling boundary states of an electrified D-brane by utilizing the prescription introduced in [29]. In section 4, we extract conserved currents directly from the boundary states proposed in section 3. We first review how to read the conserved currents for radion rolling of a bare D-brane. Then we show that conserved currents obtained directly from the boundary states constructed in section 3 agree in the supergravity limit with those obtained from the effective field theory approach. In section 5, we study radiation of the bing energy into closed string as the electrified D-brane rolls down to the NS5-brane. We then compare it with the situation for tachyon rolling of unstable D-brane. By appealing to underlying Lorentz invariance, we show that electrifying rolling D-brane is simply accounted for by taking into account of closed strings with nonzero winding number. Section 6 is devoted to conclusion and discussions for points worthy of further investigation.

Note added: after the completion of this work, we found the work [34] posted in arXiv.org, which partially overlaps with our work, in which the electrified D-brane boundary states for the rolling radion is obtained by Lorentz boosting zero-mode boundary wave function. We shall show in this work that proper treatment of the oscillator transformation based on the $\mathcal{N} = 2$ Liouville boundary states is imperative for obtaining correct conserved currents from the boundary states.

2. Prelude: effective field theory approach

We shall begin with effective field theory approach to radion rolling of electrified D-brane in the background of NS5-branes. The supergravity background of a stack of N coincident NS5-branes is given by [9, 13, 35]:

$$\begin{aligned} ds^2_{\text{string}} &= \eta_{\alpha\beta}\, dx^\alpha dx^\beta + H(x^n)\ \delta_{mn}\ dx^m dx^n\ , \\ e^{2\Phi} &= g^2_{\text{st}} H(x^n), \qquad H_{mnp} = -\epsilon^q{}_{mnp}\partial_q \Phi\ , \end{aligned} \tag{2.1}$$

where $\alpha, \beta = 0, \ldots, 5;\ \ m, n = 6, \ldots, 9$ refer to directions parallel and transverse, respectively, to the NS5-brane worldvolume and

$$H(x^n) = 1 + \frac{N\alpha'}{r^2}$$

is the harmonic function in the direction transverse to the NS5-branes. Consider a Dp-brane extended parallel to the NS5-branes. Dp-brane dynamics in the transverse space was studied in terms of Born-Infeld action in [8]. There, it was shown that absorption of D-brane to NS5-branes resembles decay of unstable D-brane in flat space via tachyon condensation. In the NS5-brane background, the string coupling becomes large and semiclassical analysis would typically break down. An interesting point, as observed in [8], is that there exists a range of D-brane energy for which a significant part of the dynamical process occurs when the string coupling is rather weak and the perturbative string theory approach would still be applicable.

2.1 Effective action and classical trajectory

We shall now extend the effective field theory analysis of [8] to electrified D-brane — a D-brane with electric flux turned on. So consider $B_{01} = -B_{10} = \varepsilon$, with all other components of $B_{\mu\nu}$ being zero. The Born-Infeld action of Dp-brane now reads:[3]

$$S_{\rm p} = -\tau_{\rm p} g_{\rm st} \int {\rm d}^{{\rm p}+1}\sigma\, e^{-\Phi} \sqrt{-\det(X^*[G+B]_{\mu\nu})}\,, \tag{2.2}$$

where $X^*[G+B]$ denotes pullback to Dp-brane worldvolume:

$$X^*[G]_{\mu\nu} = \frac{\partial X^A}{\partial \xi^\mu}\frac{\partial X^B}{\partial \xi^\nu} g_{AB} \qquad \text{and} \qquad X^*[B]_{\mu\nu} = \frac{\partial X^A}{\partial \xi^\mu}\frac{\partial X^B}{\partial \xi^\nu} b_{AB} \qquad (A, B = 0, \ldots, 9)\,.$$

We shall fix worldvolume reparametrization invariance by static gauge: $\sigma^\mu = x^\mu$. The pullback is then given by

$$A_{\mu\nu} \equiv X^*[G+B]_{\mu\nu} = g_{\mu\nu} + b_{\mu\nu} + H(R)\partial_\mu R \partial_\nu R\,.$$

[3] We denote Dp-brane worldvolume parameters as σ^μ ($\mu = 0, 1, \ldots, {\rm p}$) and tension as $\tau_{\rm p} \equiv g_{\rm st}^{-1}(2\pi)^{-{\rm p}}\alpha'^{-({\rm p}+1)/2}$.

JHEP01(2005)052

For rigid motion of a Dp-brane, denoting its transverse position as $R(t)$ and spatial volume as $V_{\rm p}$, the Born-Infeld action eq. (2.2) is reduced to

$$\begin{aligned} S_{\rm p} &= -\tau_{\rm p} V_{\rm p} \int {\rm d}t \sqrt{\frac{1-\varepsilon^2}{H(R)} - \left(\frac{{\rm d}R}{{\rm d}t}\right)^2} \\ &= -\tau_{\rm p} V_{\rm p} \int {\rm d}s \sqrt{\frac{1}{H(R)} - \left(\frac{{\rm d}R}{{\rm d}s}\right)^2} \,. \end{aligned} \tag{2.3}$$

In the second line, we recasted the action in a suggestive form by introducing "boosted time" $s \equiv \sqrt{1-\varepsilon^2}t$. We are interested in D$p$-brane dynamics primarily in the the "throat region", viz. $R \ll \sqrt{N\alpha'}$. In this region, the action eq. (2.3) is further simplified to[4]

$$S_{\rm p} = -\tau_{\rm p} V_{\rm p} \int {\rm d}t \, \frac{R}{\sqrt{N\alpha'}} \sqrt{1-\varepsilon^2 - N\alpha' \left(\frac{\rm d}{{\rm d}t} \log \frac{R}{\sqrt{N\alpha'}}\right)^2} \,. \tag{2.4}$$

Change the dynamical variable as

$$\exp\left(\frac{\phi}{\sqrt{N\alpha'}}\right) \equiv \frac{R}{\sqrt{N\alpha'}} \ll 1 \,,$$

and the action eq. (2.4) now reads

$$S_{\rm p} = -\tau_{\rm p} V_{\rm p} \int {\rm d}t \, V(\phi) \sqrt{1-\varepsilon^2 - \dot{\phi}^2}, \qquad \text{where} \qquad V(\phi) = \exp\left(\frac{\phi}{\sqrt{N\alpha'}}\right). \tag{2.5}$$

One readily finds the classical trajectory as

$$\phi(t) = -\sqrt{N\alpha'} \; \ln\left(\frac{\tau_{\rm p} V_{\rm p}}{E} \cosh \frac{\sqrt{1-\varepsilon^2}t}{\sqrt{N\alpha'}}\right), \tag{2.6}$$

where E is the conserved total energy measured in unit of the "boosted time" $s = \sqrt{1-\varepsilon^2}\, t$. Evidently, both the Born-Infeld action eq. (2.3) and the classical trajectory eq. (2.6) are simply those of Dp-brane in NS5-brane background but in terms of the "boosted time" $s = \sqrt{1-\varepsilon^2}\, t$. This statement will become clearer in the next section where we construct exact radion rolling boundary state of the Dp-brane via a chain of mapping involving Lorentz boost and T-dualities.

It is also interesting to compare the situation with the trajectory with nonzero angular momentum L [8]:

$$\phi(t) = -\sqrt{N\alpha'} \; \ln\left(\frac{\tau_{\rm p} V_{\rm p}}{E\sqrt{1-\frac{L^2}{N\alpha' E^2}}} \cosh \frac{t}{\sqrt{N\alpha'}} \sqrt{1-\frac{L^2}{N\alpha' E^2}}\right).$$

Analogy with eq. (2.6) is evident: T-dual of "internal" Lorentz-boost is replaced by "rotational" Lorentz-boost.

[4]In the following expressions, we have rescaled $\tau_{\rm p} \to \tau_{\rm p} g_{\rm st}$ and $R \to g_{\rm st}^{-1} R$.

2.2 Conserved current tensors

Conserved Noether currents provide useful probe for analyzing D-brane dynamics. For later comparison, we shall now derive energy-momentum and string current tensors for the electrified D-brane rolling along the classical trajectory eq. (2.6). Varying the Born-Infeld action eq. (2.2) with respect to background closed string fields, we obtain[5]

$$\begin{aligned} T_{\mu\nu} &\equiv 2\frac{\delta S_{\rm p}}{\delta g^{\mu\nu}} = -\frac{\tau_{\rm p} g_{\rm st}}{2} e^{-\Phi}\sqrt{-\det A}(A^{-1})_{(\mu\nu)} \\ Q_{\mu\nu} &\equiv 2\frac{\delta S_{\rm p}}{\delta b^{\mu\nu}} = -\frac{\tau_{\rm p} g_{\rm st}}{2} e^{-\Phi}\sqrt{-\det A}(A^{-1})_{[\mu\nu]}\,. \end{aligned}$$

Explicitly,

$$\begin{aligned} T_{00} &= +\frac{\tau_{\rm p}}{\sqrt{H}}\frac{1}{\sqrt{1-\varepsilon^2-H\dot{X}^m\dot{X}^m}}\,, \\ T_{11} &= -\frac{\tau_{\rm p}}{\sqrt{H}}\frac{1-H\dot{X}^m\dot{X}^m}{\sqrt{1-\varepsilon^2-H\dot{X}^m\dot{X}^m}}\,, \\ Q_{01} &= +\frac{\tau_{\rm p}}{\sqrt{H}}\frac{\varepsilon}{\sqrt{1-\varepsilon^2-H\dot{X}^m\dot{X}^m}}\,, \\ T_{ij} &= -\frac{\tau_{\rm p}}{\sqrt{H}}\sqrt{1-\varepsilon^2-H\dot{X}^m\dot{X}^m}\;\delta_{ij}\,, \qquad (i,j=2,\ldots,{\rm p}) \end{aligned}$$

and all other components vanish. In these expressions, we also suppressed delta function factors localizing the D-brane on the classical trajectory eq. (2.6). They are all conserved Noether currents.

In the near-horizon limit, from the Born-Infeld action eq. (2.5), the conserved current tensors are reduced to

$$\begin{aligned} T_{00} &= +\tau_{\rm p} e^{\frac{\phi}{\sqrt{2N}}}\frac{1}{\sqrt{1-\varepsilon^2-\dot{\phi}^2}}\,, \\ T_{11} &= -\tau_{\rm p} e^{\frac{\phi}{\sqrt{2N}}}\frac{1-\dot{\phi}^2}{\sqrt{1-\varepsilon^2-\dot{\phi}^2}}\,, \\ Q_{01} &= +\tau_{\rm p} e^{\frac{\phi}{\sqrt{2N}}}\frac{\varepsilon}{\sqrt{1-\varepsilon^2-\dot{\phi}^2}}\,, \\ T_{ij} &= -\tau_{\rm p} e^{\frac{\phi}{\sqrt{2N}}}\sqrt{1-\varepsilon^2-\dot{\phi}^2}\;\delta_{ij} \qquad (i,j=2,\ldots,{\rm p})\,. \end{aligned} \tag{2.7}$$

Reinstating delta function factors for $\phi(t)$ and thus imposing the classical trajectory eq. (2.6), we obtained the following expressions for the conserved current tensors:[6]

$$T_{00} = +\frac{E}{V_{\rm p}}\gamma\;\delta(\phi-\phi(t))\,,$$

[5] From now on, we set $\alpha' = 2$ for our convenience.

[6] Other components are deducible from those in eq. (2.7) via modified current conservation to be discussed later. See eq. (4.2).

$$
\begin{aligned}
T_{11} &= -\frac{E}{V_\mathrm{p}}\gamma\left(\mathrm{sech}^2\left(\frac{\gamma^{-1}t}{\sqrt{2N}}\right)+\varepsilon^2\tanh^2\left(\frac{\gamma^{-1}t}{\sqrt{2N}}\right)\right)\delta(\phi-\phi(t))\,,\\
Q_{01} &= +\frac{E}{V_\mathrm{p}}\ \varepsilon\ \gamma\ \delta(\phi-\phi(t))\,,\\
T_{ij} &= -\delta_{ij}\,\frac{E}{V_\mathrm{p}}\ \gamma^{-1}\mathrm{sech}^2\left(\frac{\gamma^{-1}t}{\sqrt{2N}}\right)\delta(\phi-\phi(t))\,,\\
T_{0\phi} &= +\frac{E}{V_\mathrm{p}}\ \tanh\left(\frac{\gamma^{-1}t}{\sqrt{2N}}\right)\ \delta(\phi-\phi(t))\,,\\
T_{\phi\phi} &= +\frac{E}{V_\mathrm{p}}\ \gamma^{-1}\ \tanh^2\left(\frac{\gamma^{-1}t}{\sqrt{2N}}\right)\ \delta(\phi-\phi(t))\,,\\
Q_{\phi 1} &= +\frac{E}{V_\mathrm{p}}\ \varepsilon\ \tanh\left(\frac{\gamma^{-1}t}{\sqrt{2N}}\right)\ \delta(\phi-\phi(t))\,.
\end{aligned}
\tag{2.8}
$$

Here, $\gamma \equiv 1/\sqrt{1-\varepsilon^2}$. At asymptotic future infinity, $t\to\infty$, all current components vanish except the first three, viz. T_{00}, T_{11}, Q_{01}. We readily notice that these currents consist of two parts — pressureless dust referred as "radion matter" (rolling radion counterpart to the tachyon matter) and pressure-carrying F-string dust. Indeed, in the limit $\varepsilon\to 0$, the classical trajectory eq. (2.6) and the conserved currents eq. (2.8) are reduced respectively to[7]

$$
\phi_0(t) = -\sqrt{2N}\ln\left(\frac{\tau_\mathrm{p}V_\mathrm{p}}{E}\cosh\left(\frac{t}{\sqrt{2N}}\right)\right)
$$

and

$$
\begin{aligned}
T_{00} &= +\frac{E}{V_\mathrm{p}}\delta(\phi-\phi_0(t))\,,\\
T_{0\phi} &= +\frac{E}{V_\mathrm{p}}\tanh\left(\frac{t}{\sqrt{2N}}\right)\delta(\phi-\phi_0(t))\,,\\
T_{ij} &= -\frac{E}{V_\mathrm{p}}\mathrm{sech}^2\left(\frac{t}{\sqrt{2N}}\right)\delta(\phi-\phi_0(t))\delta_{ij}\qquad (i,j=1,\ldots,\mathrm{p})\,.
\end{aligned}
$$

In this case, the pressure vanishes monotonically as $t\to\infty$, yielding a pressureless "radion matter". Then, the rest of the contribution arising at nonzero ε ought to be attributed to electifying the Dp-brane. Such two-component fluid behavior would also have been anticipated since turning on electric field is T-dual in a suitable (de)compactification limit to the Lorentz boost of D$(p-1)$-brane in a direction parallel to NS5-brane with constant velocity ε.

Conserved currents analyzed above indeed behave the same as those for unstable D-brane [28, 29], thus supporting expectation that the "geometric realization" would persist to hold to the situation where the D-brane is electified. In the next section, we shall show that the "geometric realization" can also be seen from full-fledged string theory construction in terms of relevant boundary states.

[7] This agrees with the result obtained in [17].

3. Boundary states for electrified D-brane

String theoretic account for radion rolling dynamics of electrified D-brane requires construction of an appropriate boundary state. We shall do so in this section. We first recapitulate aspects of the rolling radion boundary state of a bare D-brane constructed in [15] relevant for foregoing discussions. The construction proceeded by performing an appropriate Wick rotation of the hairpin D-brane [36, 37], which corresponds to so-called class-2 brane in the $\mathcal{N}=2$ Liouville theory [38].

3.1 Bare D-brane rolling in "throat geometry"

In the near horizon limit, stack of N NS5-branes develops so-called "throat geometry". String theory in this background is described by the superconformal field theory [11, 12, 13]

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times \mathrm{SU}(2)_{N-2} \cong \mathbb{R}^{5,1} \times \frac{[\mathbb{R}_\phi \times \mathbb{S}^1_Y] \times M_{N-2}}{\mathbb{Z}_N}, \tag{3.1}$$

where M_k denotes the $\mathcal{N}=2$ minimal model with level k and central charge $\hat{c}=k/(k+2)$, and $\mathbb{R}_\phi \times \mathbb{S}^1_Y$ denotes the $\mathcal{N}=2$ Liouville theory with $\hat{c}=1+\mathcal{Q}^2=1+\frac{2}{N}$. The $\mathbb{Z}_N$-orbifolding acts as the Gliozzi-Scherk-Olive (GSO) projection and enforces integrality of the total $\mathcal{N}=2$ U(1)-charge.

To construct the boundary state for noncompact D-brane and to enable appropriate Wick rotation into the boundary state of the sought-for radion-rolling D-brane, we consider another $\mathcal{N}=2$ Liouville system as a part of the fundamental building block. For this, we take the U(1) direction of the $\mathcal{N}=2$ Liouville theory out of the noncompact $X \in \mathbb{R}^{5,1}$ (rather than out of the SU(2) part, as usually done). This results in $\mathcal{N}=2$ superconformal algebra generated by the currents

$$\begin{aligned} T &= -\frac{1}{2}(\partial X)^2 - \frac{1}{2}(\partial\phi)^2 - \frac{\mathcal{Q}}{2}\partial^2\phi - \frac{1}{2}(\Psi^+\partial\Psi^- - \partial\Psi^+\Psi^-) \\ G^\pm &= -\Psi^\pm(i\partial X \pm \partial\phi) \mp \mathcal{Q}\partial\Psi^\pm \\ J &= \Psi^+\Psi^- - \mathcal{Q}i\partial X\,, \end{aligned} \tag{3.2}$$

where $X(z)X(0) \sim -\ln z$, $\phi(z)\phi(0) \sim -\ln z$, $\Psi^\pm(z)\Psi^\mp(0) \sim \frac{1}{z}$, $\Psi^\pm(z)\Psi^\pm(0) \sim 0$, and $\Psi^\pm = -\frac{1}{\sqrt{2}}(\psi^X \pm i\psi^\phi)$ are free fields. The $\mathcal{N}=1$ supercurrent is embedded as $G=(G^+ + G^-)/\sqrt{2}$. The linear dilaton background runs into strong string coupling singularity. We resolve the singularity by turning on an appropriate $\mathcal{N}=2$ Liouville potential. There are in fact several viable choices of the potential, all compatible with the $\mathcal{N}=2$ superconformal currents eq. (3.2). For the present system, adopting argument of [15], nonchiral Liouville potential is better suited. However, for foregoing considerations, precise form of the Liouville potential is largely irrelevant, so we will not specify the choice explicitly. Furthermore, since dependence on the cosmological constant μ is recoverable from the Knizhnik-Polyakov-Zamolodchikov (KPZ) scaling argument [39], we shall set $\mu=1$ (in a suitable unit) unless stated otherwise and avoid unnecessary complication. In the spacetime description, this originates from freedom of shifting the ϕ coordinate, and our convention is equivalent to setting $\frac{\tau_p V_p}{E}=2$.

JHEP01(2005)052

The non-BPS hairpin brane[8] in the "throat geometry" is then given by direct product of noncompact class-2 brane of the $\mathcal{N}=2$ Liouville theory and appropriate Cardy boundary state of the $\mathrm{SU}(2)_{N-2}$ sector.[9] The precise form of the latter is not important and we take, for instance, $|L=0\rangle$ corresponding to a D0-brane located at the North pole on $\mathbb{S}^3$.

D-brane's shape is determined by the Cardy boundary states of the $\mathcal{N}=2$ Liouville theory. For a non-BPS hairpin brane, we take the class-2 boundary states [38, 41]:[10]

$$|P,Q\rangle^{(\sigma)} = \int_0^\infty \mathrm{d}p \int_{-\infty}^\infty \mathrm{d}q\ \Psi^{(\sigma)}_{P,Q}(p,q)|p,q\rangle\rangle^{(\sigma)}\,,$$

labelled by (P,Q). Here, σ denotes the spin structure and the Ishibashi states $|p,q\rangle\rangle^{(\sigma)}$ are defined by the irreducible $\mathcal{N}=2$ massive characters:

$$\mathrm{ch}^{(\sigma)}(p,q;\tau,z) = e^{2\pi i\tau(\frac{p^2}{2}+\frac{q^2}{2})}e^{2\pi i\mathcal{Q}qz}\frac{\theta_{[\sigma]}(\tau,z)}{\eta(\tau)^3}\,.$$

Wave functions of the class-2 boundary states are then given by

$$\Psi^{(\sigma)}_{P,Q}(p,q) = \sqrt{2}\mathcal{Q}e^{2\pi i\frac{Qq}{\mathcal{Q}}}\cos(2\pi Pp)\frac{\Gamma\left(i\mathcal{Q}p\right)\Gamma\left(1+i\frac{2p}{\mathcal{Q}}\right)}{\Gamma\left(\frac{1}{2}+i\frac{p}{\mathcal{Q}}+\frac{q}{\mathcal{Q}}-\frac{\nu(\sigma)}{2}\right)\Gamma\left(\frac{1}{2}+i\frac{p}{\mathcal{Q}}-\frac{q}{\mathcal{Q}}+\frac{\nu(\sigma)}{2}\right)}\,, \tag{3.3}$$

where $\nu(\mathrm{NS})=0$ and $\nu(\mathrm{R})=1$. The parameter Q appears only through irrelevant phase-factor, so the class-2 boundary states are classified solely in terms of the parameter P. The simplest wave function, eq. (3.3) with $P=0$, is the one localized along the hairpin curve: $\exp(-\frac{1}{2}\mathcal{Q}\phi) = 2\cos\frac{\mathcal{Q}x}{2}$.

As proposed in [15], radion-rolling boundary states are obtained by Wick rotation of the hairpin D-brane boundary states. An important point is that the Wick-rotated momentum space wavefunction is *not* just given by naive replacement $q\to i\omega$ of eq. (3.3) but also contains nontrivial damping factor arising from noncompactness of the rolling D-brane trajectory and nontrivial choice of the configuratoin space integration contour. Applying the prescribed Wick rotation, the rolling-radion boundary state is obtained as

$$|B\rangle = \tau_p\int_0^\infty \mathrm{d}p\int_{-\infty}^\infty \mathrm{d}\omega\ \Psi^{\mathrm{NS}}(p,\omega)|p,\omega\rangle\rangle$$

where the boundary wave function $\Psi(p,\omega)$ is given by

$$\begin{aligned}\Psi^{\mathrm{NS}}(p,\omega) &= \frac{i\sqrt{2}\mathcal{Q}\sinh(\frac{2\pi p}{\mathcal{Q}})}{2\cosh[\frac{\pi}{\mathcal{Q}}(p+\omega)]\cosh[\frac{\pi}{\mathcal{Q}}(p-\omega)]}\cdot\frac{\Gamma(i\mathcal{Q}p)\Gamma(1+i\frac{2p}{\mathcal{Q}})}{\Gamma(\frac{1}{2}-i\frac{\omega}{\mathcal{Q}}+i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}+i\frac{\omega}{\mathcal{Q}}+i\frac{p}{\mathcal{Q}})}\\ &= \frac{1}{\mathcal{Q}}\frac{\sqrt{2}}{\pi}\frac{\Gamma(\frac{1}{2}+i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(1+i\mathcal{Q}p)}{\Gamma(1-\frac{2ip}{\mathcal{Q}})}\,.\end{aligned} \tag{3.4}$$

[8]This non-BPS hairpin brane has non-vanishing RR charge and was called (would-be) BPS in [8], so should be distinguished from the non-BPS brane studied in [23, 25].

[9]This is a typical example of a D-brane in the noncompact Gepner model. For a thorough study on this subject, see [40].

[10]See also [42, 36, 43, 44] for related studies.

JHEP01(2005)052

This wave function defines stringy counterpart to Dp-brane's trajectory in the supergravity approximation limit, $N = 2/\mathcal{Q}^2 \to \infty$.[11] Indeed, in this limit, Fourier-transform of the wave function eq. (3.4) is seen peaked along the trajectory:

$$e^{-\frac{1}{2}\mathcal{Q}\phi} = 2\cosh\frac{\mathcal{Q}t}{2}\,.$$

This trajectory describes rolling radion and is in full accord with the effective field theory result, eq. (2.6).

Recall that, in constructing tachyon rolling boundary states, two different approaches have been proposed. One approach is Sen's construction [1, 2] and the other one is the time-like boundary Liouville (TBL) construction [45]. The main difference between the two is that Sen's boundary state is expanded by the Ishibashi states constructed from not only the pure momentum states but also full (euclidean) $c = 1$ primaries. Our construction of radion rolling boundary state is closer in structure to the TBL approach in that all the Ishibashi states have normalizable inner products. Still, it is distinct from TBL approach in detail in that no analytic continuation of CFT parameter (such as $b \to i$ in TBL) is needed. We emphasize that this does not necessarily imply that coupling to imaginary momentum modes e^{-mT} is zero. The spirit of the Liouville theory asserts that, on the contrary, one can extract these one-point functions from analytic continuation of the real momentum Ishibashi state expansion.[12]

3.2 Electrified D-brane rolling in "throat geometry"

Consider now an electrified Dp-brane rolling down the "throat geometry" of NS5-branes. Within the effective field theory approach studied in section 2, we found that the classical trajectory of electrified Dp-brane is exactly the same as that of bare Dp-brane except "time dilation effect" $t \to \sqrt{1-\varepsilon^2}t$. It indicates that the two might be related by a Lorentz boost.

Indeed, in constructing radion rolling boundary state for an electrified Dp-brane, intuitive and technically simple way is to utilize stringy version of the Lorentz boost. Such prescription was already worked out in [29] and involves successive application of T-duality, Lorentz boost and inverse T-duality. In [29], the prescription was mainly applied to tachyon rolling of an electrified unstable Dp-brane, but the prescription is general enough and hence is equally applicable to the radion rolling of Dp-brane. In practice, the prescription can be summarized as taking the following replacement to the boundary state of a bare Dp-brane:

$$\begin{array}{lll} |0\rangle \to \gamma^{-1}|0\rangle & t \to \gamma^{-1}t & \omega \to \gamma\omega \\ \begin{pmatrix}\alpha^0\\ \alpha^1\end{pmatrix} \to \Lambda^{-1}\begin{pmatrix}\alpha^0\\ \alpha^1\end{pmatrix} & \begin{pmatrix}\psi^0\\ \psi^1\end{pmatrix} \to \Lambda^{-1}\begin{pmatrix}\psi^0\\ \psi^1\end{pmatrix} & \\ \begin{pmatrix}\overline{\alpha}^0\\ \overline{\alpha}^1\end{pmatrix} \to \Lambda\begin{pmatrix}\overline{\alpha}^0\\ \overline{\alpha}^1\end{pmatrix} & \begin{pmatrix}\overline{\psi}^0\\ \overline{\psi}^1\end{pmatrix} \to \Lambda\begin{pmatrix}\overline{\psi}^0\\ \overline{\psi}^1\end{pmatrix} & \end{array} \tag{3.5}$$

[11] Note that this limit corresponds to strong coupling limit of $\mathcal{N}=2$ Liouville theory. On the other hand, in the T-dual, $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ theory, the limit corresponds to weak coupling limit. The latter is therefore the correct description of NS5-brane "throat geometry" in the supergravity approximation.

[12] For related discussions on this point, see [46, 47, 48, 49].

where

$$\Lambda = \gamma \begin{pmatrix} 1 & +\varepsilon \\ +\varepsilon & 1 \end{pmatrix} \qquad \Lambda^{-1} = \gamma \begin{pmatrix} 1 & -\varepsilon \\ -\varepsilon & 1 \end{pmatrix} \qquad \gamma \equiv \frac{1}{\sqrt{1-\varepsilon^2}},$$

and α, ψ denote bosonic and fermionic free field oscillators. Using the prescription, we shall now construct the radion rolling boundary state of an electrified Dp-brane and show that they yield in the supergravity limit to classical trajectory and conserved currents that agree with those obtained in section 2 from the effective field theory approach.

In applying the prescription eq. (3.5), one main obstacle is that the starting theory — lorentzian $\mathcal{N}=2$ Liouville theory ($\phi, T = X^0$) coupled to free X^1 superconformal field theory — is an interacting conformal field theory. In particular, the boundary state of the former is spanned by the $\mathcal{N}=2$ Virasoro module and not by the Fock module on which the replacement eq. (3.5) is readily definable. For example, the A-type Ishibashi boundary state is expanded as:

$$|A\rangle\rangle = \Big(\cdots + \frac{i\sigma}{2h_0 - q_0} G^+_{-\frac{1}{2}} \overline{G}^+_{-\frac{1}{2}} + \frac{i\sigma}{2h_0 + q_0} G^-_{-\frac{1}{2}} \overline{G}^-_{-\frac{1}{2}} + 1 \Big) |j, q\rangle \,, \tag{3.6}$$

where $\sigma = \pm 1$ is the spin structure, $2h_0 = -j(\mathcal{Q}+j) + q^2$ and $q_0 = -q\mathcal{Q}$.[13] Utilization of the Virasoro module (instead of the Fock module) has been an important step in solving the boundary Liouville theory via the so-called modular bootstrap method [50, 38]. Determination and classification of possible boundary wave functions in the Liouville theory crucially relied on such features.

We believe this is a technical issue the physics itself gets around. Operationally, one can drop out the Liouville potential and realize the $\mathcal{N}=2$ Virasoro algebra by using the free field Fock representation eq. (3.2). This is certainly justifiable in the weak coupling end, $\phi \to +\infty$, in the "throat geometry", where the closed string modes evolve freely. With such proviso, at level-1/2, we may replace in eq. (3.6)

$$G^+_{-\frac{1}{2}} \;\to\; -(q-j)\Psi^+_{-\frac{1}{2}}\,; \qquad G^-_{-\frac{1}{2}} \;\to\; -(q+j)\Psi^-_{-\frac{1}{2}}$$

and obtain

$$|A\rangle\rangle = \left(\cdots + i\sigma \frac{(q-j)}{(q+j+\mathcal{Q})} \Psi^+_{-\frac{1}{2}} \overline{\Psi}^+_{-\frac{1}{2}} + i\sigma \frac{(q+j)}{(q-j-\mathcal{Q})} \Psi^-_{-\frac{1}{2}} \overline{\Psi}^-_{-\frac{1}{2}} + 1 \right) |j, q\rangle \,. \tag{3.7}$$

We can thus formally rewrite the $\mathcal{N}=2$ Virasoro Ishibashi states in terms of the $\mathcal{N}=2$ free-field Fock module. Having expressed rolling-radion boundary states for a bare D-brane in terms of the Fock module, we can utilize the prescription eq. (3.5) and construct rolling-radion boundary states for an electrified D-brane. Actually, as we will see in the next section, Fock module realization is quite imperative not only for extracting wave functions and classical trajectory therein but also for extracting conserved currents.

Applying the prescription eq. (3.5) to the bare D-brane boundary states eq. (3.4), we finally obtain the electrified rolling-radion boundary state

$$|B, \varepsilon\rangle = \tau_p \int_0^\infty \mathrm{d}p \int_{-\infty}^\infty \mathrm{d}\omega \;\, \Psi_\varepsilon(p, \omega) |p, \omega; \varepsilon\rangle\rangle \,,$$

[13] Roughly speaking, $|j, q\rangle = e^{j\phi + iqX}|0\rangle = e^{(-\frac{\mathcal{Q}}{2} + ip)\phi + iqX}|0\rangle$.

where the modified Ishibashi states $|p,\omega;\varepsilon\rangle\rangle$ is constructed accordingly from the prescription eq. (3.5). The wave function $\Psi_\varepsilon(p,\omega)$ is then extracted as

$$\Psi_\varepsilon(p,\omega) = \frac{i\sqrt{2}\mathcal{Q}\sinh(\frac{2\pi p}{\mathcal{Q}})}{2\cosh[\frac{\pi}{\mathcal{Q}}(p+\gamma\omega)]\cosh[\frac{\pi}{\mathcal{Q}}(p-\gamma\omega)]} \cdot \frac{\Gamma(i\mathcal{Q}p)\Gamma(1+i\frac{2p}{\mathcal{Q}})}{\Gamma(\frac{1}{2}-i\frac{\gamma\omega}{\mathcal{Q}}+i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}+i\frac{\gamma\omega}{\mathcal{Q}}+i\frac{p}{\mathcal{Q}})} \,. \tag{3.8}$$

From the zero mode part eq. (3.8) alone, we can draw variety of physics intuition.[14] For example, the zero-mode part yields information on radion-rolling trajectory of the electrified D-brane. By comparing eq. (3.8) with the first expression of eq. (3.4), we immediately find that the position space wave function of electrified D-brane is given by $\Psi_\varepsilon(\phi,t) = \Psi_0(\phi,\sqrt{1-\varepsilon^2}t)$, where $\Psi_0(\phi,t)$ is the position space wave function of bare D-brane obtained by Fourier transform of eq. (3.4). As emphasized in the previous subsection, in the supergravity limit $N\to\infty$, $\Psi_0(\phi,t)$ is localized on the radion rolling trajectory of the bare D-brane, $\Psi_0(\phi,t)\sim\delta(\phi-\phi_0(t))$. Thus, our proposed boundary wavefunction is localized in the rolling trajectory of electrified D-brane with expected "Lorentz time dilation" $t\to\gamma^{-1}t$:

$$\Psi_\varepsilon(\phi,t) = \Psi_0(\phi,\sqrt{1-\varepsilon^2}t) \sim \tau_p\delta(\phi-\phi_0(\gamma^{-1}t)) \,.$$

This trajectory agrees with the one obtained from the effective field theory analysis in section 2.

Comments are in order.

- The boundary interaction which represents the class-2 brane in the (euclidean) $\mathcal{N}=2$ Liouville theory was studied in [51, 52, 53]. In order to accommodate the constant electric field background, one needs to modify the boundary interaction accordingly. Our proposal is that the prescription eq. (3.5) to the boundary interaction is the correct one (after the appropriate Wick rotation). To all orders in worldsheet perturbation theory, the resulting boundary interaction is marginal.

- By construction, it is not apparent that the prescription eq. (3.5) is compatible with the $\mathcal{N}=2$ worldsheet superconformal symmetry. On the other hand, the $\mathcal{N}=1$ part of it, which is to be gauged, is manifestly preserved. To show this, consider a boundary state $|B\rangle$ obeying $\mathcal{N}=1$ superconformal boundary condition and a new boundary state $\mathcal{P}_0\mathbb{U}_\varepsilon|B\rangle$, where $\mathcal{P}_0$ stands for the projection operator onto zero-winding modes and $\mathbb{U}_\varepsilon$ refers to the left-right asymmetric Lorentz boost by ε. Notice that the combined operation $\mathcal{P}_0\mathbb{U}_\varepsilon$ is precisely the prescription eq. (3.5). We now prove that the boosted boundary state $\mathcal{P}_0\mathbb{U}_\varepsilon|B\rangle$ obeys the $\mathcal{N}=1$ conformal boundary condition as well:

$$(G_r + i\sigma\overline{G}_{-r})\mathcal{P}_0\mathbb{U}_\varepsilon|B\rangle = 0 \,.$$

  Since $\mathcal{P}_0$ commutes with $(G_r+i\sigma\overline{G}_{-r})$ and since the $\mathcal{N}=1$ boundary condition, in contrast to the full $\mathcal{N}=2$ superconformal symmetry, is invariant under the Lorentz

[14] Aspects of the higher-level parts will be discussed in the context of conserved currents in the next section.

boost along the $\mathbb{R}^{1,5}$ plane (longitudinal to NS5-brane), it follows that

$$\begin{aligned}(G_r + i\sigma\overline{G}_{-r})\mathcal{P}_0\mathbb{U}_\varepsilon|B\rangle &= \mathcal{P}_0\mathbb{U}_\varepsilon\mathbb{U}_\varepsilon^{-1}(G_r + i\sigma\overline{G}_{-r})\mathbb{U}_\varepsilon|B\rangle \\ &= \mathcal{P}_0\mathbb{U}_\varepsilon(G_r + i\sigma\overline{G}_{-r})|B\rangle \\ &= 0\,,\end{aligned}$$

where in the last line, we have used the property that the original boundary states $|B\rangle$ satisfy the $\mathcal{N} = 1$ conformal boundary condition. Similar consideration for the worldsheet energy-momentum tensor leads to the same conclusion. We thus see that any boundary states obtained by the prescription eq. (3.5) are indeed consistent boundary states.

4. Conserved currents of rolling radion boundary states

In understanding radion-rollong dynamics, conserved charges provide indispensible information. So, having obtained the exact boundary states, we shall now investigate such conserved charges. In section 2, the conserved charges were computed in the effective field theory approach. We shall see that, in the supergravity limit $N = \frac{2}{Q^2} \to \infty$, conserved charges extracted from exact boundary states agree with those from the effective field theory.

Again, we shall focus on D-brane dynamics at weak coupling region, where the $\mathcal{N} = 2$ Liouville potential is dropped off and the "throat geometry" is describable by $\mathcal{N} = 2$ linear dilaton conformal field theory. Expanding in terms of free Fock modules, the boundary state in the linear dilaton background $\Phi(x) = V_\mu x^\mu$ takes the form

$$|B\rangle = \tau_p \int \frac{\mathrm{d}^d\mathbf{k}}{(2\pi)^d}\Big[\widetilde{B}(\mathbf{k}) - i\sigma\widetilde{A}_{\mu\nu}(\mathbf{k})\psi^\mu_{-\frac{1}{2}}\overline{\psi}^\nu_{-\frac{1}{2}} + \cdots\Big]|\mathbf{k}\rangle_0\,, \qquad \mathbf{k}_\mu \in \mathbb{R}^d\,.$$

Energy-momentum tensor of the boundary state is then given by (see e.g. [54])

$$T_{\mu\nu}(x) = -e^{-V_\mu x^\mu}\Big(A_{\mu\nu}(x) + B(x)\eta_{\mu\nu}\Big)\,. \tag{4.1}$$

in the convention of the Fourier transformation:

$$f(x) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} e^{ik_\mu x^\mu}\widetilde{f}(k)\,.$$

It is readily seen that the energy-momentum tensor eq. (4.1) satisfies the shifted conservation laws:

$$\partial^\mu T_{\mu\nu} = \frac{1}{2}V_\nu U\,, \qquad \frac{1}{4}U = e^{-V_\mu x^\mu}B\,.$$

4.1 Conserved currents for bare D-brane

We shall first examine conserved currents for the bare D-brane case and compare them with the effective field theory results. Expand the boundary state in free field Fock states:

$$|B\rangle = \tau_{\mathrm{p}}\Big[B - i\sigma(A_{(\mu\nu)} + C_{[\mu\nu]})\psi^\mu_{-\frac{1}{2}}\overline{\psi}^\nu_{-\frac{1}{2}} + \cdots\Big]|0\rangle\,, \tag{4.2}$$

where $A_{\mu\nu}$, $C_{\mu\nu}$ refer to symmetric and antisymmetric irreducible parts. With the linear dilaton background along the Liouville direction, the energy-momentum tensor is given by

$$T_{\mu\nu} = -\tau_{\rm p}\ e^{\frac{1}{2}\mathcal{Q}\phi}(A_{\mu\nu} + \eta_{\mu\nu}\ B)\,,$$

while the string current tensor is given by

$$Q_{\mu\nu} = -\tau_{\rm p}\ e^{\frac{1}{2}\mathcal{Q}\phi}C_{\mu\nu}\,.$$

We expect them to reduce to the effective field theory results of section 2 in the supergravity limit, $N = \frac{2}{\mathcal{Q}^2} \to \infty$.

The first term B in eq. (4.2) involves no Fock oscillator, so it is extractible from the boundary wave function eq. (3.4) by making Fourier transform. A useful integral formula is

$$\frac{1}{\mathcal{Q}}\int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}e^{-i\omega t}\frac{\Gamma(\frac{1}{2}+i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})}{\Gamma(1-\frac{2ip}{\mathcal{Q}})} = \left[2\cosh\left(\frac{t\mathcal{Q}}{2}\right)\right]^{\frac{2ip}{\mathcal{Q}}-1}. \tag{4.3}$$

In the supergravity limit $N = \frac{2}{\mathcal{Q}^2} \to \infty$, the $\Gamma(1+i\mathcal{Q}p)$ factor in eq. (3.4) may be dropped off. The result is

$$\begin{aligned} B(\phi,t) &= \frac{1}{\mathcal{Q}}\int_{-\infty}^{\infty}\frac{{\rm d}\omega{\rm d}p}{(2\pi)^2}e^{ip\phi-i\omega t}\frac{\sqrt{2}}{\pi}\frac{\Gamma(\frac{1}{2}+i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})}{\Gamma(1-\frac{2ip}{\mathcal{Q}})} \\ &= \frac{1}{\cosh(\frac{t\mathcal{Q}}{2})}\frac{\sqrt{2}}{2\pi}\delta\left(\phi+\frac{2}{\mathcal{Q}}\log\left(2\cosh\frac{\mathcal{Q}t}{2}\right)\right) \\ &\equiv \frac{\sqrt{2}}{2\pi}B_0(\phi,t)\,. \end{aligned} \tag{4.4}$$

The numerical factor $\frac{\sqrt{2}}{2\pi}$ is absorbable by rescaling the string coupling constant $g_{\rm st}$. The (00)-component of the second term $A_{(\mu\nu)}$ in eq. (4.2) is obtainable from eq. (3.7) and eq. (3.4). We first Wick-rotate, expand the free fermion oscillators into components

$$\Psi^{\pm}\overline{\Psi}^{\pm} = \frac{1}{2}\left(\psi^X\overline{\psi}^X \pm i\psi^\phi\overline{\psi}^X \pm i\psi^X\overline{\psi}^\phi - \psi^\phi\overline{\psi}^\phi\right),$$

and extract the $\psi^0_{-\frac{1}{2}}\overline{\psi}^0_{-\frac{1}{2}}$ component from the boundary states. In the supergravity limit $N = \frac{2}{\mathcal{Q}^2} \to \infty$, the factor $\Gamma(1+i\mathcal{Q}p)$ can be dropped off. With Fock module realization $j \to ip - \mathcal{Q}/2$ and Wick rotation $q \to i\omega$, we find that

$$\begin{aligned} \frac{2\pi}{\sqrt{2}}\widetilde{A}_{00}(p,\omega) &= -(-i)^2\frac{1}{\mathcal{Q}}\frac{\Gamma(\frac{1}{2}+i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})}{\Gamma(1-\frac{2ip}{\mathcal{Q}})}\left(\frac{i\omega-ip+\frac{\mathcal{Q}}{2}}{i\omega+ip+\frac{\mathcal{Q}}{2}}+\frac{i\omega+ip-\frac{\mathcal{Q}}{2}}{i\omega-ip-\frac{\mathcal{Q}}{2}}\right) \\ &= -\frac{1}{\mathcal{Q}}\frac{\Gamma(\frac{1}{2}+1+i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}-1-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})}{\Gamma(1-\frac{2ip}{\mathcal{Q}})} - \\ &\quad -\frac{1}{\mathcal{Q}}\frac{\Gamma(\frac{1}{2}-1+i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}+1-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})}{\Gamma(1-\frac{2ip}{\mathcal{Q}})} \\ &\equiv \widetilde{A}_0(p,\omega)\,. \end{aligned}$$

JHEP01(2005)052

In the first line, the first sign factor $(-)$ is due to relative sign convention between eq. (3.7) and eq. (4.2), while the second sign factor $(-i)^2$ arises from the Wick rotation from A_{XX} to A_{00}. Fourier transforming the amplitude by formally substituting $\omega \to \omega \pm i\mathcal{Q}$ into each term of eq. (4.3) and modifying the integration contour,[15] we find

$$\begin{aligned} A_{00}(\phi,t) &= \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega \mathrm{d}p}{(2\pi)^2} e^{ip\phi - i\omega t} \widetilde{A}_{00}(p,\omega) \\ &= -\frac{\cosh(t\mathcal{Q})}{\cosh(\frac{t\mathcal{Q}}{2})} \frac{\sqrt{2}}{2\pi} \delta\left(\phi + \frac{2}{\mathcal{Q}} \log\left(2\cosh\frac{\mathcal{Q}t}{2}\right)\right) \\ &\equiv \frac{\sqrt{2}}{2\pi} A_0(\phi,t)\,. \end{aligned} \tag{4.5}$$

We also find that $A_{11}(\phi,t)$ equals to the zero-mode wave function $B(\phi,t)$ since it comes from the flat Dirichlet Ishibashi states. Then, independent components of the energy-momentum tensor are[16]

$$\begin{aligned} T_{00}(\phi,t) &= -\tau_{\mathrm{p}}\, e^{\frac{1}{2}\mathcal{Q}\phi}(A_{00} - B) = \delta(\phi - \phi_0(t)) \\ T_{ij}(\phi,t) &= -\tau_{\mathrm{p}}\, e^{\frac{1}{2}\mathcal{Q}\phi}(A_{11} + B) = -\mathrm{sech}^2\left(\frac{\mathcal{Q}t}{2}\right)\delta_{ij}\delta(\phi - \phi_0(t)) \end{aligned}$$

where $\phi_0(t)$ is the trajectory in the supergravity limit

$$\phi_0(t) = -\frac{2}{\mathcal{Q}} \log\left(2\cosh\frac{\mathcal{Q}t}{2}\right). \tag{4.6}$$

The result is in agreement with the result obtained from effective field theory approach in section 2.2.[17]

Of particular interest is exploring stringy effects present at finite N and $\mathcal{Q}$. Repeating the analysis but retaining the factor $\Gamma(1 + i\mathcal{Q}p)$ in the boundary wave function, we now find that

$$B(\phi,t) = \frac{\sqrt{2}}{\pi\mathcal{Q}\left(2\cosh\frac{\mathcal{Q}t}{2}\right)^{\frac{2}{\mathcal{Q}^2}+1}} \exp\left[-\frac{\phi}{\mathcal{Q}} - \frac{e^{-\frac{\phi}{\mathcal{Q}}}}{\left(2\cosh\frac{\mathcal{Q}t}{2}\right)^{\frac{2}{\mathcal{Q}^2}}}\right]. \tag{4.7}$$

We thus learn that the classical trajectory is now smeared to the profile:

$$\frac{1}{\mathcal{Q}} \frac{1}{\left(2\cosh\frac{\mathcal{Q}t}{2}\right)^{\frac{2}{\mathcal{Q}^2}}} \exp\left[-\frac{\phi}{\mathcal{Q}} - \frac{e^{-\frac{\phi}{\mathcal{Q}}}}{\left(2\cosh\frac{\mathcal{Q}t}{2}\right)^{\frac{2}{\mathcal{Q}^2}}}\right].$$

[15] This is a formal manipulation since the direct calculation yields a divergent result. We should regard it as an analytic continuation or a suitable Wick rotation from the corresponding euclidean calculation, where no divergence arises. See [15] on this issue.

[16] All other components are obtained from the conservation law of the energy momentum tensor. In the boundary conformal field theory language, this is equivalent to the statement that the boundary states preserve (half of) the $\mathcal{N} = 1$ superconformal symmetry.

[17] Recall that we have chosen shift of the ϕ coordinate so that $\tau_{\mathrm{p}} V_{\mathrm{p}}/E = 2$.

JHEP01(2005)052

In turn, the energy-momentum tensor at finite $N = \frac{2}{\mathcal{Q}^2}$ is given by

$$T_{00}(\phi, t) = \frac{1}{\mathcal{Q}} \left(\frac{e^{-\frac{\mathcal{Q}\phi}{2}}}{2\cosh\frac{\mathcal{Q}t}{2}} \right)^{\frac{2}{\mathcal{Q}^2}-1} \cdot \exp\left[-\left(\frac{e^{-\frac{\mathcal{Q}\phi}{2}}}{2\cosh\frac{\mathcal{Q}t}{2}} \right)^{\frac{2}{\mathcal{Q}^2}} \right], \tag{4.8}$$

replacing the delta function localized energy density at supergravity limit into smeared distribution in ϕ-direction. One can check readily that, after integrating over ϕ, the total energy is a constant of motion. The results are in full accord with expectation that stringy effects would smear out the classical trajectory to a distribution of height and width set by $\mathcal{Q} = \sqrt{2/N}$.

To gain intuition how the stringy effects smear out the trajectory, consider the energy density $T_{00}(\phi, t)$ and examine its behavior both near asymptotic past and future $t \to \pm\infty$ and around time-symmetric turning point $t \sim 0$. For the trajectory in the supergravity limit, $\phi(t)$ evolves as

$$\phi_0(t) \simeq \begin{cases} -|t| \mp \frac{2}{\mathcal{Q}} e^{-|t|\mathcal{Q}} + \cdots & \text{for } t \gg \pm\frac{2}{\mathcal{Q}} \\ -\frac{2}{\mathcal{Q}}\log 2 + \frac{\mathcal{Q}}{4}|t|^2 + \cdots & \text{for } |t| \sim 0 \end{cases} .$$

For the stringy profile at finite $N = \frac{2}{\mathcal{Q}^2}$, eq. (4.8) clearly reveals that the profile is a Poisson-type distribution. Maximum of the energy density distribution is now located at

$$\frac{e^{-\frac{\mathcal{Q}\phi}{2}}}{2\cosh\frac{\mathcal{Q}t}{2}} = 1 - \frac{\mathcal{Q}^2}{2},$$

so it deviates from the trajectory $\phi_0(t)$ in the supergravity limit by $\Delta\phi \equiv (\phi(t) - \phi_0(t)) \simeq \mathcal{Q}$. In fact, for $\phi = \phi_0(t)$, each factor in round bracket in eq. (4.8) is reduced to unity. The variance $\Delta\phi$ of the distribution may be estimated more accurately by expanding $\phi(t)$ around $\phi_0(t)$ in each factor of round brackets in eq. (4.8). We find that

$$T_{00}(\phi, t) \simeq T_{00}\Big|_{\max} \left(1 - \frac{1}{2}N(N-1)(\Delta\phi)^2 + \cdots \right),$$

and hence estimate the variance of the energy-density distribution as

$$\Delta\phi \simeq \sqrt{\frac{2\mathcal{Q}^2}{2 - \mathcal{Q}^2}}. \tag{4.9}$$

The result bears two interesting implications. First, unlike rolling-tachyon boundary states for unstable D-brane for which the boundary state exhibits a universal variance of the boundary wave function set by the string scale $\sqrt{\alpha'}$, rolling-radion boundary state exhibits $\mathcal{Q}$-dependent variance. We interpret this as a feature originating from the fact that D-brane is now placed in curved background of "throat geometry" rather than flat Minkowski spacetime, as considered for unstable D-brane. Second, both the energy density eq. (4.8) and the variance eq. (4.9) indicate that $\mathcal{Q}^2 = 2$ is a special point. Notice, however, that this highly stringy regime does not correspond to a NS5-brane background. Rather, being

$N = 1$, this describes a conifold background. Indeed, it has been recurrently noted that closed string sector of $\mathcal{N} = 2$ Liouville theory, equivalently, $\mathcal{N} = 2$ $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ theory changes drastically across this selfdual point $\mathcal{Q}^2 = 2$. This phenomenon has to do with the fact that the chiral $\mathcal{N} = 2$ Liouville potential violates the Seiberg bound. Our observation is that open string sector of these theories also changes drastically across the same selfdual point. In the next section, we shall also discover another manifestation of such effect in the context of closed string radiation out of radion rolling.

4.2 Conserved currents for electrified D-brane

We shall now extend analysis of conserved currents to the electrified D-brane. After turning on boundary interaction for the electric field, the rolling-radion boundary state is modified according to the prescription eq. (3.5). One readily finds that the prescription eq. (3.5) gives rise to the following transformation to the boundary states:

$$\begin{aligned}
B(\phi, t) &= +\gamma^{-1}\, B_0(\phi, \gamma^{-1}t) \\
A_{00}(\phi, t) &= +\gamma\, A_0(\phi, \gamma^{-1}t) - \varepsilon^2\, \gamma B_0(\phi, \gamma^{-1}t) \\
C_{01}(\phi, t) &= +\gamma\, \varepsilon\, (A_0(\phi, \gamma^{-1}t) - B_0\,(\phi, \gamma^{-1}t)) \\
A_{11}(\phi, t) &= +\gamma\, B_0(\phi, \gamma^{-1}t) - \varepsilon^2\, \gamma A_0(\phi, \gamma^{-1}t) \\
A_{ij}(\phi, t) &= +\gamma^{-1}\, B_0(\phi, \gamma^{-1}t)\, \delta_{ij}\,,
\end{aligned}$$

where now i, j run for space transverse to the boost direction. Here, $A_0(\phi, t)$ and $B_0(\phi, t)$ are components of the boundary wave functions for bare D-brane given in eq. (4.5) and eq. (4.4), respectively. In the supergravity limit $N \to \infty$ and $\mathcal{Q} \to 0$, using B_0, A_0 obtained in the previous subsection, it is straightforward to check that the boundary wavefunction is reduced to the result of effective field theory analysis in section 2.2.

By construction, the energy-momentum tensor transforms covariantly under the Lorentz boost and hence is always conserved. Still, it would be illuminating to check this by direct computations. So, consider $T_{0\phi} = e^{\frac{1}{2}\mathcal{Q}\phi} A_{0\phi}$, viz. momentum density along ϕ-direction. It suffices to examine it in the supergravity limit $N = \frac{2}{\mathcal{Q}^2} \to \infty$:

$$\begin{aligned}
\frac{2\pi}{\sqrt{2}} \widetilde{A}_{0\phi}(p, \omega) &= \frac{1}{\mathcal{Q}} \frac{\Gamma(\frac{1}{2} + i\frac{\gamma\omega}{\mathcal{Q}} - i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2} - i\frac{\gamma\omega}{\mathcal{Q}} - i\frac{p}{\mathcal{Q}})}{\Gamma(1 - \frac{2ip}{\mathcal{Q}})} \left(\frac{i\gamma\omega - ip + \frac{\mathcal{Q}}{2}}{i\gamma\omega + ip + \frac{\mathcal{Q}}{2}} - \frac{i\gamma\omega + ip - \frac{\mathcal{Q}}{2}}{i\gamma\omega - ip - \frac{\mathcal{Q}}{2}} \right) \\
&= -\frac{1}{\mathcal{Q}} \frac{\Gamma(1 + \frac{1}{2} + i\frac{\gamma\omega}{\mathcal{Q}} - i\frac{p}{\mathcal{Q}})\Gamma(-1 + \frac{1}{2} - i\frac{\gamma\omega}{\mathcal{Q}} - i\frac{p}{\mathcal{Q}})}{\Gamma(1 - \frac{2ip}{\mathcal{Q}})} + \\
&\quad + \frac{1}{\mathcal{Q}} \frac{\Gamma(-1 + \frac{1}{2} + i\frac{\gamma\omega}{\mathcal{Q}} - i\frac{p}{\mathcal{Q}})\Gamma(1 + \frac{1}{2} - i\frac{\gamma\omega}{\mathcal{Q}} - i\frac{p}{\mathcal{Q}})}{\Gamma(1 - \frac{2ip}{\mathcal{Q}})}\,.
\end{aligned}$$

Fourier transforming back to (ϕ, t)-space,

$$\begin{aligned}
A_{0\phi}(\phi, t) &= \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega \mathrm{d}p}{(2\pi)^2}\, e^{ip\phi - i\omega t}\, A_{0\phi}(p, \omega) \\
&= -\frac{\sinh(\gamma^{-1}\mathcal{Q}t)}{\cosh(\frac{\gamma^{-1}\mathcal{Q}t}{2})} \frac{\sqrt{2}}{2\pi} \delta\left(\phi + \frac{2}{\mathcal{Q}} \log\left(\cosh\frac{\gamma^{-1}\mathcal{Q}t}{2}\right)\right)\,,
\end{aligned}$$

we find it agrees with the effective field theory result eq. (2.8). It demonstrates that the prescription eq. (3.5) manifestly retains conservation of the energy-momentum tensor (as well as other boundary state currents) and BRST symmetry of the closed string equation of motion in the presence of the boundary state as a source.

Again, it is of particular interest to explore stringy effects present at finite N and $\mathcal{Q}$ and to see how the electrification intertwine with such stringy effects. Proceeding analogously to the bare D-brane case, we find that the energy density now reads

$$T_{00}(\phi, t) = \frac{\gamma}{\mathcal{Q}} \left(\frac{e^{-\frac{\mathcal{Q}\phi}{2}}}{2\cosh \frac{\mathcal{Q}\gamma^{-1}t}{2}} \right)^{\frac{2}{\mathcal{Q}^2}-1} \cdot \exp\left[- \left(\frac{e^{-\frac{\mathcal{Q}\phi}{2}}}{2\cosh \frac{\mathcal{Q}\gamma^{-1}t}{2}} \right)^{\frac{2}{\mathcal{Q}^2}} \right].$$

This clearly indicates that the trajectory eq. (2.6) in the supergravity limit $N = \frac{2}{\mathcal{Q}^2} \to \infty$ is now replaced by the same Poisson-like distribution as for the bare D-brane case. Note that, as expected, Lorentz covariance rendered the energy density distribution get modified by overall γ Lorentz boost and time dilation $t \to \gamma^{-1}t$. Thus, electrifying the D-brane left no effect on the functional form of the energy profile. Moreover, variance of the distribution $\Delta\phi$ is Lorentz invariant, viz. the same for both the bare D-brane and the electrified one. The value $\mathcal{Q}^2 = 2$ is still the selfdual critical point. This fact further supports the interpretation that the selfdual critical point has intrinsically to do with closed string sector of the $\mathcal{N} = 2$ Liouville and $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ theories.

It is straightforward to extract other conserved currents coupled to higher oscillator modes out of the boundary states. As discussed in [29], the prescription eq. (3.5) renders components of conserved currents coupled to the oscillator along electric field strength tensor directions are Lorentz boosted by appropriate powers of the Lorentz factor γ. This effect then serves as a useful mnemonic for understanding the boundary states and higher conserved currents therein [30]. It is also possible to consider magnetifying D-brane by Lorentz rotation. Since the procedure is also prescribed in [29] and is sufficiently trivial, we shall not repeat it here.

5. Radiative decay of electrified D-brane

Having constructed exact boundary states of an electrified D-brane, we now investigate aspects of D-brane's radiative decay as it rolls down the "throat geometry" and eventually dissolve into the NS5-brane. Final state of the process is a bound-state of the electrified D-brane and the NS5-brane. Since the bound-state is of non-threshold type, we expect that excess binding energy would be released partly into closed string emission and, if present, partly into some sort of "radion matter".

Again, by the sequence of T-duality, Lorentz boost and inverse T-duality, in an appropriate (de)compactification limit of the boost direction, the radiative decay process of an electrified D-brane would be related Lorentz covariantly to that of a bare D-brane. This line of reasoning is elementary and intuitive, so we shall first spell out the argument explicitly and then check it by direct computations.

5.1 Bare versus electrified: Lorentz boost and T-dual

Consider a bare Dp-brane rolling down the "throat geometry". Since the motion is rigid and homogeneous, we will compactify a worldvolume direction common to the Dp-brane and the NS5-brane into a circle of radius R. We shall then take $R \to 0$ limit. T-dualizing along the circle direction, we now have D$(p-1)$-brane on a dual circle of radius $2/R$, which is then decompactified in the $R \to 0$ limit. Let us now boost the D$(p-1)$-brane and measure the average number density of the closed strings emitted for a fixed transverse mass M, $\langle N(M)\rangle / V_{\rm p}$:

$$\frac{\langle N(M)\rangle}{V_{\rm p}} = \sum\!\!\!\!\!\!\int \mathrm{dVol}\frac{1}{2\omega}|\Psi(\omega)|^2 \,,$$

where the integral is over the phase-space of emitted quanta and $\Psi(\omega)$ is the transition amplitude. Obviously, this is the quantity independent of choice of the reference frame, so it ought to be expressible as an integral over final state phase-space variables either in the rest frame or in the boosted frame.

To demonstrate the Lorentz invariance of $\langle N(M)\rangle / V_{\rm p}$, begin with the phase-space integral of the Dp-brane in the rest frame. The on-shell energy is given by $E_p = \sqrt{p^2 + \mathbf{k}^2 + M^2}$ where p is the momentum along ϕ-direction, $\mathbf{k}$ is the momentun along $(5-p)$ relative transverse directions to the Dp-brane in $\mathbb{R}^5$ and M is a fixed transverse mass. Expressing it in terms of spectral variable, the average number density for the NS-sector is given by

$$\begin{aligned}
\frac{\langle N(M)\rangle}{V_{\rm p}} &= \int \mathrm{d}p\mathrm{d}\mathbf{k}\,\frac{1}{2E_p}\frac{1}{\cosh(\frac{2\pi}{\mathcal{Q}}E_p) + \cosh(\frac{2\pi p}{\mathcal{Q}})}\frac{2\sinh(\frac{2\pi p}{\mathcal{Q}})}{\sinh(\pi p \mathcal{Q})} \\
&= \int \mathrm{d}p\,\mathrm{d}\mathbf{k}\,\mathrm{d}E\,\frac{\theta(E)\delta(E^2 - p^2 - \mathbf{k}^2 - M^2)}{\cosh(\frac{2\pi}{\mathcal{Q}}E) + \cosh(\frac{2\pi p}{\mathcal{Q}})}\frac{2\sinh(\frac{2\pi p}{\mathcal{Q}})}{\sinh(\pi p \mathcal{Q})}\,.
\end{aligned}$$

Now, decompose the spatial momentum as $\mathbf{k} = (\ell, \mathbf{k}_\perp)$ and consider the boosted frame along the ℓ-direction by velocity ε. The phase-space variables of the two frames are then related by the Lorentz transformations:

$$E = \gamma(E' - \varepsilon\ell')\,, \qquad \ell = \gamma(\ell' - \varepsilon E')\,.$$

So, the average number density is now expressible in the boosted frame as

$$\begin{aligned}
\frac{\langle N(M)\rangle}{V_{\rm p}} &= \int \mathrm{d}p\mathrm{d}\mathbf{k}_\perp\mathrm{d}\ell'\mathrm{d}E'\,\frac{\theta(E')\,\delta(E'^2 - \ell'^2 - p^2 - \mathbf{k}_\perp^2 - M^2)}{\cosh\left[\frac{2\pi}{\mathcal{Q}}\gamma(E' - \varepsilon\ell')\right] + \cosh(\frac{2\pi p}{\mathcal{Q}})}\frac{2\sinh(\frac{2\pi p}{\mathcal{Q}})}{\sinh(\pi p \mathcal{Q})} \\
&= \int \mathrm{d}p\mathrm{d}\mathbf{k}_\perp\mathrm{d}\ell'\,\frac{1}{2E'_p}\frac{1}{\cosh\left[\frac{2\pi}{\mathcal{Q}}\gamma(E'_p - \varepsilon\ell')\right] + \cosh(\frac{2\pi p}{\mathcal{Q}})}\frac{2\sinh(\frac{2\pi p}{\mathcal{Q}})}{\sinh(\pi p \mathcal{Q})}\,,
\end{aligned}$$

where $E'_p = \sqrt{p^2 + (\mathbf{k}_\perp^2 + \ell'^2) + M^2}$. In the T-dual picture, this last expression is interpretable as the average number of closed string emitted out of electrified Dp-brane *in the*

$R \to 0$ *limit*. Explicitly, it is given by

$$\frac{\langle N(M)\rangle}{V_{\rm p}} = \lim_{R\to 0}\frac{R}{2}\sum_{w=-\infty}^{+\infty}\int {\rm d}p{\rm d}\mathbf{k}_\perp \frac{1}{2E'_p}\frac{1}{\cosh\left[\frac{2\pi}{\mathcal{Q}}\gamma(E'_p - \frac{1}{2}\varepsilon R w)\right] + \cosh(\frac{2\pi p}{\mathcal{Q}})}\frac{2\sinh(\frac{2\pi p}{\mathcal{Q}})}{\sinh(\pi p\mathcal{Q})}, \tag{5.1}$$

where we T-dualized the longitudinal momentum ℓ' to the winding $\frac{1}{2}Rw$. Notice that the compactification limit $R \to 0$ is imperative since, for finite R, the Lorentz symmetry is broken in the original picture and the effect of the electric flux appears in the T-dualized picture. The expression involves sum over all winding quantum numbers, w. However, for any finite value of the radiation energy E'_p, it is evident from the kinematics that w would take virtually a continuous value in the limit $R \to 0$. Thus, in the compactification limit, one can evaluate the phase-space sum and integral via saddle-point approximations. It is obvious from eq. (5.1) that the saddle-point for ω is located around a value set by the electric field ε times the energy E'_p. It is large for a large value of the transverse mass M. Notice also that the integrand depends nontrivially on the winding quantum number w and also that the integrand at zero winding $w = 0$ is exponentially suppressed compared to that at the saddle point.

In addition to the average number density, one also would like to extract spectral shape of the emitted closed strings. Convolution of the spectral shape is measured by moments of the energy emitted $\langle E^N(M)\rangle/V_{\rm p}$ and also by moments of the winding number emitted $\langle \omega^N(M)\rangle/V_{\rm p}$ $(N = 1, 2, \ldots)$. They are extracted by weighing $E^N_{p,w}$ or ω^N into the phase-space integral of eq. (5.1):

$$\begin{aligned}
\frac{\langle E^N(M)\rangle}{V_{\rm p}} &= \lim_{R\to 0}\frac{R}{2}\sum_{w=-\infty}^{+\infty}\int {\rm d}p{\rm d}\mathbf{k}_\perp \frac{1}{2E'_p}(E'_p)^N \times \\
&\qquad\times \frac{1}{\cosh\left[\frac{2\pi}{\mathcal{Q}}\gamma(E'_p - \frac{1}{2}\varepsilon R w)\right] + \cosh(\frac{2\pi p}{\mathcal{Q}})}\frac{2\sinh(\frac{2\pi p}{\mathcal{Q}})}{\sinh(\pi p\mathcal{Q})} \\
\frac{\langle w^N(M)\rangle}{V_{\rm p}} &= \lim_{R\to 0}\frac{R}{2}\sum_{w=-\infty}^{+\infty} w^N \int {\rm d}p{\rm d}\mathbf{k}_\perp \frac{1}{2E'_p} \times \\
&\qquad\times \frac{1}{\cosh\left[\frac{2\pi}{\mathcal{Q}}\gamma(E'_p - \frac{1}{2}\varepsilon R w)\right] + \cosh(\frac{2\pi p}{\mathcal{Q}})}\frac{2\sinh(\frac{2\pi p}{\mathcal{Q}})}{\sinh(\pi p\mathcal{Q})}.
\end{aligned} \tag{5.2}$$

These observables transform covariantly under the Lorentz boost, so those measured in the rest frame can be related to those measured in the boosted frame.

5.2 Direct computation: winding versus no-winding

Though the Lorentz covariance argument makes it clear that the emitted closed strings carry nonzero winding quantum number w set by the electric field ε, for those who would like to see it explicitly, we shall evaluate spectral observables eq. (5.2) directly using the exact boundary states constructed in the previous section. In particular, we shall show explicitly that radiation of closed strings with zero winding is exponentially suppressed due to the presence of the electric field. This phenomenon is the geometric counterpart of the same phenomenon for the rolling tachyon [28, 55, 54, 56] qualitatively, but there is some quantitative difference as we will see.

The starting point of our argument is the boundary states of the electrified D-brane

$$|B,\varepsilon\rangle = \int_0^\infty \mathrm{d}p \int_{-\infty}^\infty \mathrm{d}\omega\, \Psi_\varepsilon(p,\omega)|p,\omega;\varepsilon\rangle\rangle\,,$$

where Ishibashi states $|p,\omega;\varepsilon\rangle\rangle$ can be constructed by successive operations of T-duality and Lorentz boost as in section 3. The boundary wave function $\Psi_\varepsilon(p,\omega)$ represents the coupling to zero modes and, for NS sector,

$$\Psi^{\mathrm{NS}}_\varepsilon(p,\omega) = \frac{i\sqrt{2}\mathcal{Q}\sinh(\frac{2\pi p}{\mathcal{Q}})}{2\cosh[\frac{\pi}{\mathcal{Q}}(p+\gamma\omega)]\cosh[\frac{\pi}{\mathcal{Q}}(p-\gamma\omega)]}\cdot\frac{\Gamma(i\mathcal{Q}p)\Gamma(1+i\frac{2p}{\mathcal{Q}})}{\Gamma(\frac{1}{2}-i\frac{\gamma\omega}{\mathcal{Q}}+i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}+i\frac{\gamma\omega}{\mathcal{Q}}+i\frac{p}{\mathcal{Q}})}\,.$$

We focus on the NS sector wave function, since other sectors are readily obtainable by the spectral flow. As was discussed in [15], the emission rate does not show any qualitative difference among these different spin structures (at least in the UV region).

For a fixed transverse mass M, the average number density of emitted closed string is given by

$$\begin{aligned}\frac{\langle N(M)\rangle}{V_{\mathrm{p}}} &= \int \mathrm{d}\mathbf{k}\int_0^\infty \frac{\mathrm{d}p}{2E_p}\gamma^{-2}|\Psi^{\mathrm{NS}}_\varepsilon(p,E_p)|^2\\ &= \int \mathrm{d}\mathbf{k}\int_0^\infty \frac{\mathrm{d}p}{2E_p}\gamma^{-2}\frac{2\sinh\left(\frac{2\pi p}{\mathcal{Q}}\right)}{\left[\cosh\left(\frac{2\pi\gamma E_p}{\mathcal{Q}}\right)+\cosh\left(\frac{2\pi p}{\mathcal{Q}}\right)\right]\sinh(\pi\mathcal{Q}p)}\,,\end{aligned}\tag{5.3}$$

where $E_p = \sqrt{p^2+\mathbf{k}^2+M^2}$ is the on-shell energy of the emitted closed string states. Likewise, N-th moment of the energy emitted $\langle E^N(M)\rangle/V_{\mathrm{p}}$ is obtainable by weighing the integrand by E_p^N.

These spectral moments involve sum over all final states.[18] It is evident from eq. (5.3) that the phase-space integral involves sum over the infinite massive states and hence may cause ultraviolet catastrophe. To see if the divergence actually occurs, we need to sum over these massive modes for large values of M. Evaluating it via the saddle point method, we find that

$$\begin{aligned}\frac{\langle N(M)\rangle}{V} &\simeq \int \mathrm{d}\mathbf{k}\int_0^\infty \frac{\mathrm{d}p}{M}\gamma^{-2}e^{(\frac{2\pi}{\mathcal{Q}}-\pi\mathcal{Q})p-\frac{2\pi}{\mathcal{Q}}\gamma\sqrt{p^2+\mathbf{k}^2+M^2}}\\ &\simeq e^{-2\pi M\gamma\sqrt{1-\frac{\mathcal{Q}^2}{4}+\varepsilon^2(\frac{1}{\mathcal{Q}}-\frac{\mathcal{Q}}{2})^2}}\,.\end{aligned}\tag{5.4}$$

Now, density of states of the emitted closed string for a given mass M is the same as that of the open string. For large M, it grows as $n(M)\sim M^{-3}e^{2\pi M\sqrt{1-\frac{\mathcal{Q}^2}{4}}}$ [57]. So, summing

[18] One might observe that coupling to higher oscillator modes are different level by level, which could invalidate our naive use of only the zero mode boundary wave function. However this can be avoided by taking a suitable gauge. Alternatively, one may evaluate the imaginary part of the one-loop amplitude in the Minkowski signature, from the emission rate may be extracted via the optical theorem [54, 56]. Physically, these two prescriptions should yield identical result.

over the transverse mass M, the average number density of all closed strings emitted is estimated as

$$
\begin{aligned}
\frac{\langle N\rangle}{V_{\rm p}} &= \int_0^\infty \mathrm{d}M e^{2\pi M\sqrt{1-\frac{\mathcal{Q}^2}{4}}} \int \mathrm{d}\mathbf{k} \int_0^\infty \frac{\mathrm{d}p}{M^4}\,\gamma^{-2} e^{\left(\frac{2\pi}{\mathcal{Q}}-\pi\mathcal{Q}\right)p-\frac{2\pi}{\mathcal{Q}}\gamma\sqrt{p^2+\mathbf{k}^2+M^2}} \\
&\simeq \gamma^{-2}\int^\infty \mathrm{d}L\, L^{\frac{5-p}{2}-3} e^{-\pi L(\gamma-1)} \\
&\simeq \gamma^{-2}(\gamma-1)^{\frac{p-1}{2}}\,, \qquad (5.5)
\end{aligned}
$$

where in the second line we have introduced a dummy 'radial' variable $L^2 = (2/\mathcal{Q})^2(M^2 + \mathbf{k}^2 + p^2)$ and performed the angular integration first. The last integration is ultraviolet finite. We also note that all the spectral moments are infrared finite since the boundary wave function in the NS-sector does not have a pole in the $p, E \to 0$ limit.

It would be illuminating to compare the result eq. (5.5) with that for the decay of an unstable D-brane via tachyon rolling in the flat spacetime. In the latter situation, the transition probability is given by $|\Psi_\varepsilon(\omega)|^2 \sim \frac{1}{\sinh^2 \pi\gamma\omega}$. It results in the average number density as

$$
\frac{\langle N\rangle}{V} \simeq \int^\infty \mathrm{d}M\, e^{-2\pi M(\gamma-1)}\,.
$$

Notice that this is manifestly ultraviolet finite but may have infrared divergence. Apparently, effects of electrifying the D-brane is qualitatively the same but quantitatively different — response to ε is somewhat different in the spectral weight in eq. (5.4). This is attributed to nontrivial saddle point of the integrand in eq. (5.4). In contrast, the average total number density of closed strings eq. (5.5) exhibits similar behavior as that for rolling tachyon case. However, notice that since the contribution from the higher level is exponentially suppressed in saddle-point approximation, the change of summation into integration is not trustful in the evaluation of the average total number density.[19]

Several comments are in order.

- For bare D-brane decay, spectral density of the closed strings emitted was independent of the number of NS5-branes $N = 2/\mathcal{Q}^2$. This ensured that, regardless of the number of NS5-branes, the spectral moments of emitted closed string are exactly same as those of tachyon rolling in flat spacetime. However, such 'geometric equivalence' is lost at quantitative level once the D-brane is electrified.

- Although it is outside the scope of our interest, we have exactly the same behavior as for the tachyon rolling if we formally set $\mathcal{Q} = \sqrt{2}$. As pointed out earlier, this case corresponds to the decay of the rolling D-brane in a conifold background, not in the NS5-brane background. If we formally extend $\mathcal{Q}$ below the selfdual point, eqs. (5.4), (5.5) is absolutely ultraviolet convergent even for the bare D-brane case, $\varepsilon = 0$. Given that the Hagedorn density of state makes sense only for $\mathcal{Q} < 2$, the relevant range for the convergence would be $\sqrt{2} \le \mathcal{Q} \le 2$.

[19] This is to be contrasted against the consideration of [34].

- Our result implies that negligible number of the closed string and negligible portion of the Dp-brane energy is emitted to zero-winding states. Of course, this is as expected from the Lorentz covariance argument in the previous subsection (See also [30]). In the case of the rolling tachyon, [33] also argued that, after compactification, all the energy of electrified Dp-brane is actually radiated away into closed strings with nonzero winding quantum number. See also [58] for an explicit check to this effect.

The last point is what we already explained intuitively in the previous subsection by appealing to the Lorentz invariance, so we shall now supplement it by direct computations. Compactify along the direction of the electric field to a circle of radius R. Then the average number density for a fixed mass M is now modified to

$$\frac{\langle N(M)\rangle}{V_{\rm p}} \simeq \sum_{w=-\infty}^{+\infty} \int {\rm d}\mathbf{k} \int_0^\infty \frac{{\rm d}p}{M} \gamma^{-2} e^{\left(\frac{2\pi}{\mathcal{Q}} - \pi \mathcal{Q}\right)p - \frac{2\pi}{\mathcal{Q}}\gamma\left(\sqrt{p^2+\mathbf{k}^2+M^2+(\frac{1}{2}wR)^2} - \frac{1}{2}\varepsilon R w\right)} \tag{5.6}$$

by summing over the winding mode contributions ($w = 0$ sector reduces to eq. (5.4)). For large value of M, this sum over the winding modes can be evaluated by the saddle-point method. The result is

$$\frac{\langle N(M)\rangle}{V_{\rm p}} \simeq \int {\rm d}\mathbf{k} \int_0^\infty \frac{{\rm d}p}{M^{\frac{1}{2}}} e^{\left(\frac{2\pi}{\mathcal{Q}} - \pi \mathcal{Q}\right)p - \frac{2\pi}{\mathcal{Q}}\sqrt{p^2+\mathbf{k}^2+M^2}} . \tag{5.7}$$

Notice that dependence on ε in the exponent has disappeared after saddle-point sum over the winding quantum number w. Integrations over p, $\mathbf{k}$ and finally over M can be done exactly the same way as in [15]. This leads to power-like ultraviolet catastrophe for the average number density:

$$\frac{\langle N\rangle}{V_{\rm p}} \simeq \int^\infty {\rm d}M \; M^{-\frac{p-1}{2}-1} ,$$

or for higher spectral moments of the energy or the winding number. Notice further that the dependence on $\mathcal{Q}$ in eq. (5.7) has disappeared after integration over $p, \mathbf{k}$.

The main point we would like to bring out is that proper account of the winding modes recovers universal feature shared by both the tachyon rolling and the radion rolloing of D-branes. This result asserts that the closed string emitted mainly consists of the highly winding ones. As discussed in the previous subsection, this result is also what one would expect from elementary consideration of Lorentz covariance of spectral observables. The result also partially answers the puzzle mentioned in the first comment: after the inclusion of the winding modes, quantitative difference between the rolling tachyon and the rolling D-brane disappeared simply because spectral observables no longer depend on $\mathcal{Q}$ or ε. We emphasize again that dependence on $\mathcal{Q}$ and ε disappeared as a consequence of sum over the winding quantum number.

6. Conclusion and discussion

In this paper, we studied rolling radion dynamics of electrified D-brane in the NS5-brane background. We confirmed that "geometric realization" of tachyon rolling in terms of radion rolling continues to hold to situations where the D-brane is electrified. We constructed exact boundary states of rolling radion and compared the results with the effective field theory analysis. In the supergravity limit $N = 2/\mathcal{Q}^2 \to \infty$, we confirmed that the two approaches fully agree with each other.

For bare D-brane, it was shown that the decay of rolling radion is the same as that of the rolling tachyon *irrespective of* the total number of NS5-brane $\mathcal{Q} = \sqrt{2/N}$. In this context, it is worthwhile pointing out that the effective action agrees with each other for a specific number of the NS5-brane — $\mathcal{Q} = 1/\sqrt{2}$ for the bosonic rolling tachyon and $\mathcal{Q} = 1$ for the supersymmetric rolling tachyon.

After electrifying the D-brane, we found that exponential suppression of the spectral density of the emitted closed string, which is qualitatively analogous to the rolling tachyon dynamics of electrified D-brane, actually begins to *depend* on the value of $\mathcal{Q}$, or the number of the NS5-branes. This can be understood as a quantitative difference between the rolling D-brane and the rolling tachyon dynamics. It would be interesting to give a holographic dual explanation of this fact (in the context of little string theory) or to study distinguishing role of the deformed conifold background where the dependence on ε exactly coincides with the rolling tachyon problem. However, we also found that this quantitative difference may be an artifact of the noncompact space: after proper inclusion of the winding modes, the universal property of the decaying process is recovered. This completely agrees with the situation in the rolling tachyon.

To conclude, we would like to address a natural question on the fate of the radion rolling for electrified D-brane. Our analysis shows that conserved currents consist of two parts — the pressureless "tachyon (radion) matter" and the fundamental string fluid, just as in the case of the rolling tachyon. From the geometrical viewpoint, on the other hand, we expect that the rolling D-brane will make a bound state with the NS5-brane. Thus it would be interesting to understand what happens to the fundamental string charge, in addition to the Ramond-Ramond charge, originally carried by the D-brane. To answer this nonperturbative question, neither the effective field theory analysis nor the boundary states analysis seems to be adequate. At the least, it calls for quantum counterpart of the boundary states. On the other hand, the open string completeness conjecture (see [16] and references therein) might indicate that our analysis has already given a clue to this issue. The dual little string theory description or the M-theoretic consideration should also play a relevant role here.

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